

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

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**B.Tech.****(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13  
MATHEMATICS—I***Time : 3 Hours**Total Marks : 100***SECTION—A**

1. All parts for this question are compulsory :  $(2 \times 10 = 20)$
- (a) Find the 8<sup>th</sup> derivative of  $x^2 e^x$ .
- (b) If  $x^2 = au + bv$ ,  $y^2 = au - bv$ , then find  $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v$ .
- (c) Find the stationary points of  
 $f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$ .
- (d) If  $x = u(1 + v)$ ,  $y = v(1 + u)$ , then find the Jacobian of  $u, v$  with respect to  $x, y$ .
- (e) Reduce the matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$  into normal form.
- (f) Prove that the matrix  $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$  is unitary.
- (g) Evaluate  $\int_0^1 \int_0^{x^2} x e^y dx dy$ .
- (h) Evaluate  $\Gamma(-3/2)$ .
- (i) Find the value of  $m$  if  $\vec{F} = m\hat{i} - 5y\hat{j} + 2z\hat{k}$  is a solenoidal vector.

- (j) Find the unit normal at the surface  $z = x^2 + y^2$  at the point  $(1, 2, 5)$ .

### SECTION—B

2. Attempt any **three** parts of the following : (3×10=30)

- (a) If  $y = \left(x + \sqrt{1+x^2}\right)^m$ , then find the  $n^{\text{th}}$  derivative of  $y$  at  $x = 0$ .
- (b) Find the maximum and minimum distance of the point  $(1, 2, -1)$  from the sphere  $x^2 + y^2 + z^2 = 24$ .
- (c) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

- (d) Evaluate  $\iiint_V (ax^2 + by^2 + cz^2) dx dy dz$  where  $V$  is the region bounded by  $x^2 + y^2 + z^2 \leq 1$ .
- (e) Verify Gauss's divergence theorem for the function  $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$  over unit cube.

### SECTION—C

Attempt any **two** parts from each question of this section. All questions are compulsory. [(2×5)×5=50]

3. (a) State and prove Euler's theorem for homogeneous functions.
- (b) Expand  $f(x, y) = e^x \tan^{-1} y$  in powers of  $(x - 1)$  and  $(y - 1)$  upto two terms of degree 2.

- (c) If  $z = f(x, y)$  where  $x = e^u \cos v$ ,  $y = e^u \sin v$ , prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = e^{-2u} \left[ \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2 \right].$$

4. (a) If  $x + y + z = u$ ,  $y + z = uv$ ,  $z = uvw$ , then find  $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ .
- (b) The two sides of a triangle are measured as 50 cm and 70 cm, and the angle between them is  $30^\circ$ . If there are possible errors of 0.5% in the measurement of the sides and 0.5 degree in that of the angle, find the maximum approximate percentage error in measuring the area of the triangle.
- (c) Show that  $u = y + z$ ,  $v = x + 2z^2$ ,  $w = x - 4yz - 2y^2$  are not independent. Find the relation between them.
5. (a) Test the consistency and hence, solve the following set of equations :

$$10y + 3z = 0,$$

$$3x + 3y + 2z = 1,$$

$$2x - 3y - z = 5,$$

$$x + 2y = 4.$$

- (b) Using elementary transformations, find the rank of the following matrix :

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

- (c) Examine the following vectors for linearly dependent and find the relation between them, if possible :

$$X_1 = (1, 1, -1, 1), X_2 = (1, -1, 2, -1), X_3 = (3, 1, 0, 1).$$

6. (a) Prove that :  $\sqrt{\pi} \Gamma(2n) = 2^{2n-1} \Gamma(n) \Gamma\left(n + \frac{1}{2}\right)$ ,

where  $n$  is not a negative integer or zero.

(b) Change the order of integration and hence evaluate

$$\int_0^{\infty} \int_0^y y e^{-y^2/x} dx dy.$$

(c) Find the area of the region occupied by the curves  $y^2 = x$  and  $y^2 = 4 - x$ .

7. (a) Show that the vector field  $\vec{F} = yz\hat{i} + (zx + 1)\hat{j} + xy\hat{k}$  is conservative. Find its scalar potential. Also find the work done by  $\vec{F}$  in moving a particle from  $(1, 0, 0)$  to  $(2, 1, 4)$ .

(b) Prove that :

$$\text{Curl}(\vec{F} \times \vec{G}) = \vec{F} \text{ div } \vec{G} - \vec{G} \text{ div } \vec{F} + (\vec{G} \cdot \nabla)\vec{F} - (\vec{F} \cdot \nabla)\vec{G}.$$

(c) If  $\text{div} [f(\vec{r})\vec{r}] = 0$ , where  $\vec{r}$  is the position vector of a point  $(x, y, z)$ , then find  $f(\vec{r})$ .